THE PRESERVATION OF GEOMETRIC PROPERTIES BY DISKRETE NONAUTONOMIC INVERSE DYNAMICAL SYSTEMS UNDER THE TOPOLOGICAL CONJUGATIONS

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S.F. Kolyada in [1] put out some questions about preservation of geometric properties by dynamical systems under the topological conjugation. We have investigated the preservation different geometric properties of projections by dynamical systems. The topological conjugation is such homeomorphism which is realized by commutative diagrams for projections as $\pi \circ p_n = q_n \circ \pi$. When π is not homeomorphism but it is surjective map and when it is realized by the commutative diagrams for the projections $\pi \circ p_n = q_n \circ \pi$ therefore it named as semiconjugation.

Theorem 1. *Such geometrical properties are preserved under the topological conjugation as (1) thin homotopic equalence, (2) Z_n – set property, (3) the property of projection to be almost homeomorphism, (4) semicontinuous from below and semicontinuous from above, (5) the diskrete cell - approximation property, (6) the property of uniformly local K - connected - (ULC^k), (7) soft of projections, (8) the approximative soft of projections , (9) the approximative stratification by Gurevich, (10) Z - approximation soft property, (11) strong C - universality of projections.*

Theorem 2. *Such geometrical properties as $Helder(\alpha)$ [2] for projections is preserved under the nonexpansive semiconjugation by factor-map.*

We remember the definitions which used in theorem 1. Soft map T means that if for each closed $B \subset A$ and for each g, h such that diagram $T \circ g = h \circ i$ is commutative than exists $\varphi : A \to X$ such that will be commutative following two diagrams 1) $g = \varphi \circ i$, 2) $h = T \circ \varphi$. So T is defined as approximative soft if for each covering $\omega \in Cov(X)$ and for each closed B belong to arbitrary A and for each maps $g : B \to X_{i+1}$ and $h : A \to X_i$ the condition $T \circ g = h \circ i$ follow that exists map $\psi : A \to X_{i+1}$ such that will be commutative two diagrams (1) $\psi \circ i = g$, (2) $(T \circ \psi, h) < \omega$. We prove for instance that strong C - universality for map is invariant under the topological conjugation between dynamical systems. The map $\varphi : X \to Y$ is defined as SCU-map (strong C-universality) if for each closed $B \subset A \in C$ for each enclosing $f : B \to X$ and for each map $g : A \to Y$ such that diagram $\varphi \circ f = g|_B$ is commutative or $\varphi \circ f = g \circ i$ follow that exists the enclosing $F : A \to X$ which breaks big diagram onto two commutative triangle diagrams (1) $F \circ i = f$, (2) $\varphi \circ F = g$. We note, that if we change the word "enclosing" onto word "map" we have obtained the definition of C-soft map. We take each enclosing $\varphi : B \to Y$ and we take any map $G : A \to W$. Let such diagram $S \circ \varphi = G \circ i$ is commutative. We construct the map $f : B \to X$ by formula $f = \pi^{-1}(\varphi)$ and we construct $g = \pi^{-1}(G)$. We will to prove commutativity $T(f) = g(i)$. Really $T(f) = T \circ \pi^{-1}(\varphi)$. On the other hand $g(i) = \pi^{-1} \circ G(i)$ and we have equality $T \circ \pi^{-1}(\varphi) = \pi^{-1} \circ G(i)$. And so we use of the condition topological conjugation $\pi^{-1} \circ S = T \circ \pi^{-1}$. Put it under the beforehand equality $T \circ \pi^{-1}(\varphi) = \pi^{-1}(S) \circ \varphi$ or $T(f) = \pi^{-1} \circ S(\varphi)$. But $S(\varphi) = G(i)$ and put it under the beforehand equality $T(f) = \pi^{-1} \circ G(i) = g(i)$.

We have obtained following equality $T(f) = g(i)$. So we used the definition that map T have property strong C-universality t.i. exists enclosing $F : A \to X$

which breaks the diagram $T(f) = g(i)$ onto two commutative triangle diagrams (1) $F \circ i = f$, (2) $T \circ F = g$. So, we construct the map $\Phi : A \to X$ by the formula $\Phi = \pi(F)$. So we will to prove that map Φ breaks the diagram $S(\varphi) = G(i)$ onto two commutative triangles (1) $\Phi(i) = \varphi$ and (2) $S \circ \Phi = G$. Really $\Phi(i) = \pi \circ F(i) = \pi(f) = \varphi$, therefore $\Phi(i) = \varphi$. So $S \circ \Phi = S \circ \pi(F)$ under the condition of topological conjugation it equal $\pi \circ T(F) = \pi(g) = G$. It finish the proof.

References

[1] S.F. Kolyada *Topological dynamic.* Doctor thesis 2004. Inst. Math. NAN Ukraine, 2004.

[2] Yoav Benyamini and Yoram Lindenstrauss *Geometric Nonlinear Functional Analysis.* AMS, Colloquium Publications, Volume 48

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