DETECTING HILBERT MANIFOLDS AMONG HOMOGENEOUS SPACES

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Answering the fifth problem of D.Hilbert, A. Gleason [2] and D. Montgomery, L. Zippin [3], proved that each locally contractible locally compact topological group G is a Lie group, hence a manifold modeled on an Euclidean space $\mathbb{R}^n$. In [1] T.Dobrowolski and H.Toruńczyk extended this result and proved that each Polish ANR-group G is a Hilbert manifold, that is, a manifold modeled on a Hilbert space.

In the talk we shall address the problem of detecting Hilbert manifolds among homogeneous spaces of the form $G/H = \{xH : x \in G\}$ where H is a closed subgroup of a topological group G. Let us define a subgroup H of G to be *balanced* if for every neighborhood $U \subset G$ of the neutral element $e \in G$ there is a neighborhood $V \subset G$ of e such that $HV \subset UH$. It is easy to see that each subgroup of a SIN-group is balanced.

Theorem. *Let G be a group with a complete left-invariant metric and H be a closed balanced LC^∞-subgroup of G. The quotient space G/H is a Hilbert manifold if and only if it is an ANR.*

This theorem shows that for SIN-groups G the following open problem has affirmative solution.

Open problem. *Let G be a Polish ANR-group and H be a closed subgroup of G such that the quotient map $q : G \to G/H$ is a locally trivial bundle. Is G/H a Hilbert manifolds?*

References

[1] T. Dobrowolski and H. Toruńczyk, *Separable complete ANR's admitting a group structure are Hilbert manifolds*, Topology Appl. **12** (1981), 229–235.

[2] A.M. Gleason, *Groups without small subgroups*, Ann. Math. **56** (1952), 193–212.

[3] D. Montgomery and L. Zippin, Topological Transformation Groups, Interscience New York, 1955; Reprint of the 1955 Original, R.E. Kriger Publ. Co., Huntingto, N.Y., 1974.

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