BOUNDEDNESS OF THE L-INDEX IN DIRECTION FOR ENTIRE FUNCTIONS WITH 'PLANE' ZEROS

A.I. BANDURA

Let L is a positive continuous function in $\mathbb{C}^n$ and for every $\eta > 0$ next inequality is true $0 < \inf\{\inf\{\lambda(z,t,t_0) : |t-t_0| \le \frac{\eta}{L(z+t_0\mathbf{b})}\} : z \in \mathbb{C}^n, t_0 \in \mathbb{C}\} \le \sup\{\sup\{\lambda(z,t,t_0) : |t-t_0| \le \frac{\eta}{L(z+t_0\mathbf{b})}\} : z \in \mathbb{C}^n, t_0 \in \mathbb{C}\} < +\infty$, where $\mathbf{b} \in \mathbb{C}^n$, $\lambda(z,t,t_0) = \frac{L(z+t\mathbf{b})}{L(z+t\mathbf{b})}$. Let $Q^n_{\mathbf{b}}$ denote a class of these function $L(z)$. An entire function of $F(z)$, $z \in \mathbb{C}^n$, is called (see [1]) function of bounded L-index in the direction of $\mathbf{b} \in \mathbb{C}^n$, if there exists $m_0 \in \mathbb{Z}_+$ such that for $m \in \mathbb{Z}_+$ and every $z \in \mathbb{C}^n$ performs inequality:

$$\frac{1}{m!L^m(z)}\left|\frac{\partial^m F(z)}{\partial \mathbf{b}^m}\right| \le \max\left\{\frac{1}{k!L^k(z)}\left|\frac{\partial^k F(z)}{\partial \mathbf{b}^k}\right| : 0 \le k \le m_0\right\},$$

where $\frac{\partial^0 F(z)}{\partial \mathbf{b}^0} = F(z)$, $\frac{\partial F(z)}{\partial \mathbf{b}} = \sum\limits_{j=1}^{n} \frac{\partial F(z)}{\partial z_j} b_j$, $\frac{\partial^k F(z)}{\partial \mathbf{b}^k} = \frac{\partial}{\partial \mathbf{b}}(\frac{\partial^{k-1} F(z)}{\partial \mathbf{b}^{k-1}}), k \ge 2$.

Let F — entire function in $\mathbb{C}^n$ of p genus with 'plane' zeros ([2]):

$$F(z) = \prod_{k=1}^{\infty} g(\langle z, a^k|a^k|^{-2}\rangle, p),\ g(u,p) = (1-u)\exp\left\{u + \frac{u^2}{2} + \cdots + \frac{u^p}{p}\right\}, (1)$$

where $a^k \in \mathbb{C}^n$, is a sequence of p genus: $\sum_{k=1}^{\infty} 1/|a^k|^{p+1} < +\infty$, $\sum_{k=1}^{\infty} 1/|a^k|^p = +\infty$ and $\langle a, b\rangle = \sum\limits_{j=1}^{n} a_j \overline{b}_j$ for $a, b \in \mathbb{C}^n$. Denote $n(r) = \sum_{|a^k|<r} 1$.

Theorem 1. *If* $\frac{|a^k|^{p+1}}{k} \nearrow \infty$ $(k \to \infty)$, $a^k_j = m_j|a^k|$ *for all* k, $j = 1, \ldots, n$, $m = (m_1, \ldots, m_n)$, $L(|\langle z, m\rangle|) \in Q^n_{\mathbf{b}}$, $n(r)\ln n(r) = O(rL(r))$ *and*

$$r^{p-1}\sum_{l=1}^{n(r)} \frac{1}{|a^k|^p} + r^p \sum_{k=n(r)+1}^{\infty} \frac{1}{|a^k|^{p+1}} = O(l(r)), r \to +\infty,$$

then product (1) is a function of bounded L-index in direction $\mathbf{b}$.

References

[1] Bandura A. I., Skaskiv O. B. *Entire function of bounded L-index in direction* Matem. Stud. **27** (1) (2007), 30-52 (in Ukrainian).

[2] Papush D. E. *On the growth of entire functions with 'plane' zeros* // Theory of functions, functional analysis and its application. **48 (1987), 117–125 (in Russian).**

Department informatics and informational technologies, Precarpathian law institute of Lviv state university of internal affairs, Ivano-Frankivs'k, str. National'noyi Hvardiyi 3, Ukraine

E-mail address: andriybandura@i.ua