ON EQUIVALENT STRICTLY $G-$CONVEX RENORMINGS

N. BOYKO

Uniform $G-$convexity of Banach spaces is a recently introduced [1] natural generalization of convexity and of complex convexity. In this talk we present results on equivalent strictly $G-$convex renormings of different spaces.

Let $G \subset L(X)$. Let us call the *modulus of $G-$convexity* of X the following function:

$$\delta_X^G(t) = \inf_{||x||=||y||=1} \{\sup\{||x + \varepsilon Ty|| : T \in G\} - 1\}.$$

A space X is called *$G-$convex*, if $\delta_X^G(\varepsilon) \geq 0$ for all $\varepsilon > 0$, and X is said to be *strictly $G-$convex*, if $\sup_{T\in G} \|x + \varepsilon Ty\| > 1$ for arbitrary $x, y \in S_X$ and $\varepsilon > 0$. It is well-known that the space l_∞ can be renormed strictly convexly. And it is proved in [2] that if Γ is uncountable, then $l_\infty(\Gamma)$ cannot be renormed strictly convexly. We study an analogous question for the notion of strict $G-$convexity.

Theorem 1. *Let Γ is an uncountable set, G is a separable bounded operator family, then the space $l_\infty(\Gamma)$ cannot be renormed strictly $G-$convexly.*

Theorem 2. *Let G is a finite group, $\sum_{i=1}^n T_i = 0$, X admits strictly $G-$convex norm $\|\cdot\|$, Y is an invariant under G subspace of X. Then an arbitrary equivalent strictly $G-$convex norm $|\cdot|$ on subspace Y can be extended to an equivalent strictly $G-$convex norm on space X.*

A particular case of uniform G-convexity is the well-known complex convexity, when $G = \{e^{i\theta} I : \theta \in (0, 2\pi]\}$ or, which is equivalent, when $G = \{I, -I, iI, -iI\}$.

Corollary. *X admits strictly complex convex norm $\|\cdot\|$. Then an arbitrary equivalent strictly complex convex norm $|\cdot|$ on subspace Y can be extended to an equivalent strictly complex convex norm on space X.*

References

[1] Boyko N., *On assigning operator coefficients to the series terms*, Kharkov National University Vestnik **826**58 (2008), 197–210 (in Russian)

[2] Diestel J., *G*eometry of Banach Spaces - Selected Topics, Springer-Verlag,– 1975.

[3] Kadets V., Boyko N., *Uniform $G-$convexity for vector-valued L_p spaces*, Serdica Mathematical Journal **35**(2009), 1–14

[4] Wee-Kee Tang, *O*n the extinsion of rotund norms, Manuscripta math. **91** (1996), 73–82

Department of Mechanics and Mathematics, Kharkov National University, Kharkov, pl. Svobody 4, Ukraine

E-mail address: `maletska.nata@gmail.com`