CLOSED 1-FORMS WITH ISOLATED ZEROS ON NONORIENTABLE CLOSED SURFACES

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Let M be a nonorientable closed surface of genus p, ω be a closed 1-form on M and $N(\omega)$ be a set of zeros of ω. A curve γ in M which doesn't have zeros is called an integral curve of ω if it is a local level of function f such that $\omega = df$.

For each enough small neighborhood $U(z)$ of the point z curve passing through z breaks the $U(z)$ into two parts: positive $\{v : f(v) - f(z) > 0\}$ and negative $\{v : f(v) - f(z) < 0\}$.

Closed 1-forms ω_1 and ω_2 on M are called trajectory equivalent if there exists a homeomorphism $h : M \to M$ which maps zeros to zeros and curves to curves. And h is called a trajectory equivalence. If h maps positive parts of small neighborhoods to positive parts and negative parts to negative parts then h is called the topologically equivalence and the appropriate 1-forms are topologically equivalent. We combine positive parts to positive subdomains and negative to negative subdomains.

A zero of 1-form is called an isolated zero if there exists a neighborhood which doesn't have others zeros. An integral curve $\gamma : \mathbb{R} \to M$ is called recurrent if $\gamma \subset \{z \in M : \exists \{t_n\} \to \pm\infty, \gamma(t_n) \to z, n \to \infty\}$. We deal with closed 1-forms with isolated zeros.

For every closed 1-form ω we construct the graph $G(\omega)$ imbedded in surface M.

We consider the Poincare rotation number, the homotopy rotation class [1] and the orbit of homotopy rotation class [2].

Theorem 1. *Let M be a nonorientable closed surface of genus p and closed 1-forms ω_1 and ω_2 are given on M. ω_1 and ω_2 are topologically equivalent if and only if they satisfy the following conditions:*

1) for $G(\omega_1) \neq \emptyset$ and $G(\omega_2) \neq \emptyset$ the following conditions hold:

- *the exists a homeomorphism $f : M \to M$ such that its restriction to $G(\omega_1)$ is an isomorphism of the graphs $G(\omega_1)$ and $G(\omega_2)$;*
- *the domains restricted by edges of the graph $G(\omega_1)$ are mapped to the domains restricted by the corresponding image of edges of the graph $G(\omega_2)$;*
- *positive subdomains are mapped to positive subdomains and negative to negative subdomains.*

2) for each domain of $M \backslash G(\omega_i), i = \overline{1,2}$ which contains at least one non closed recurrent curve the following conditions hold:

- *the numbers of rotation of the curves of ω_1 and ω_2 are sodimentional for orientable domain of genus 1 and nonorientable domain of genus 3;*
- *the exists a pair of non closed recurrent separatrices of ω_1 and ω_2 whose homotopy rotation classes are sodimentional for orientable domain of genus more than 1;*
- *the exists a pair of non closed recurrent separatrices of closed 1-forms ω_1 and ω_2 whose orbits of homotopy rotation classes are the same for nonorientable domain of genus more than 3.*

References

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