TOPOLOGY OF CONJUGACY CLASSES OF MAPS

T.V. BUDNYTSKA

We consider the problem of classification of affine maps up to topological conjugacy.

Let $\mathbb{F}^n$ be the vector space $\mathbb{R}^n$ or $\mathbb{C}^n$, $n \geq 1$.

The maps $f,\, g : \mathbb{F}^n \to \mathbb{F}^n$ are said to be *topologically conjugate* (written $f \overset{t}{\sim} g$), if there exists a homeomorphism $h : \mathbb{F}^n \to \mathbb{F}^n$ such that

$$g = h \circ f \circ h^{-1}.$$

An element $x \in \mathbb{F}^n$ is called a *fixed point* of $f : \mathbb{F}^n \to \mathbb{F}^n$ if $f(x) = x$.

A map $f : \mathbb{F}^n \to \mathbb{F}^n$ of the form

$$f(x) = Ax + b$$

is called an *affine map*, where the matrix $A \in \mathbb{F}^{n\times n}$ is called the *linear part* of affine map and $b \in \mathbb{F}^n$.

Theorem 1. *Let $f,\, g : \mathbb{F}^n \to \mathbb{F}^n$, $f(x) = Ax + b$, $g(x) = Cx + d$ be affine maps and $\mathbb{F}^n = \mathbb{R}^n$ or $\mathbb{C}^n$, $n \geq 1$.*

1. *If each of f and g has at least one fixed point, then $f \overset{t}{\sim} g$ if and only if their linear parts are topologically conjugate.*

2. *If f and g have no fixed points and $n = 1$ or 2, then $f \overset{t}{\sim} g$ if and only if* $\det A$ *and* $\det C$ *are either simultaneously equal to 0 or simultaneously different from 0.*

References

[1] T.V. Budnytska, *Topological conjugacy classes of affine maps*, // arXiv:0812.4921.

Department of Geometry, Kyiv National Taras Shevchenko University, Kyiv, Volodymyrska st., 64, Ukraine

E-mail address: Budnitska_T@ukr.net