A NONHOMOGENEOUS POLARIZATION FORMULA

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A given n-linear symmetric mapping can be uniquely expressed by the n-homogeneous polynomial using the *polarization formula*, which is one of the fundamental results in the theory of polynomials and polylinear mappings. The purpose of the talk is to give an analogue of the polarization formula for nonhomogeneous polinomials and analytic maps.

The polarization formula has various representations, in particular using the generalized Rademacher functions. For every natural number $n \geq 2$ the generalized Rademacher functions $S_j^{[n]}(t)$ are defined inductively as follows (see [1], [2]). Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be the complex n-th roots of unity. For $j = 1, \dots, n$ let $I_j = \left(\frac{j-1}{n}, \frac{j}{n}\right)$ and $I_{j_1 j_2}$ denote the j_2-th open subinterval of length $\frac{1}{n^2}$ of $I_{j_1 j_2}$ $(j_1, j_2 = 1, \dots, n)$. Proceeding like this, we can define the interval $j_1 j_2 \dots j_k$ for any k. Now $S_1^{[n]}(t)$: $[0, 1] \to \mathbb{C}$ is defined by setting $S_1^{[n]}(t) = \alpha_j$ for $t \in I_j$, where $1 < j < n$. In general, $S_k^{[n]}(t) = \alpha_j$ if t belongs to the subinterval $I_{j_1 j_2 \dots j_k}$ where $j_k = j$. Also we set $S_1^{[1]}(t) \equiv 1$.

Let us denote

$$\Pi_k(P)(x_1, \dots, x_k) = \frac{1}{k!} \int_0^1 \left(S_1^{[k]}\right)^{k-1}(t) \dots \left(S_k^{[k]}\right)^{k-1}(t) \times$$

$$\times P\left(S_1^{[k]}(t)x_1 + \dots + S_k^{[k]}(t)x_k\right) dt$$

and let us introduce a set $\mathbb{N}_i \subset \mathbb{N}$ by the way: $\mathbb{N}_i = \{p = p_1 p_2 \dots p_i : p_1 < p_2 < \dots < p_i \text{ are prime numbers}\}$.

Theorem 1. *Let $P = P_0 + \dots + P_n$ be a polynomial of degree n and A_k be an k-linear mapping such that $P_k(x) = A_k(x, \dots, x)$ for some $1 \leq k \leq n$. Then*

$$A_k(x_1, \dots x_k) = \Pi_k(P)(x_1, \dots, x_k) + \sum_{i=1}^{r} (-1)^i \sum_{\mathbb{N}_i \ni p \leq r} \Pi_{pk}(P)(x_1^p, \dots, x_k^p),$$

where $r = [\frac{n}{k}]$.

Theorem 2. *Let A_k be a k-linear symmetric mapping corresponding to k-homogeneous component f_k of entire mapping of bounded type f. Then*

$$A_k(x_1, \dots x_k) = \Pi_k(f)(x_1, \dots, x_k) + \sum_{i=1}^{\infty} (-1)^i \sum_{p \in \mathbb{N}_i} \Pi_{pk}(f)(x_1^p, \dots, x_k^p),$$

where $\mathbb{N}_i = \{p = p_1 p_2 \dots p_i : p_1 < p_2 < \dots < p_i$ are prime numbers$\}$.

References

[1] R.M. Aron and J. Globevnik, *Analytic functions on c_0*, Revista Matematica, Madrid. **2** (1989), 27–34

[2] R.M. Aron, M. Lacruz, R.A. Ryan, and A.M. Tonge, *The generalized Rademacher Functions*, Note Math. **12** (1992), 15–22

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