EXPONENTIAL TYPE VECTORS OF COMPLEX DEGREES OF POSITIVE OPERATORS

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Let A be a positive operator with dense domain $\mathcal{C}^1$ in complex Banach space X. It means that $(-\infty, 0] \in \rho(A)$, where $\rho(A)$ is resolvent set of A and there exists $c > 0$ such that $\|(A-\lambda I)^{-1}\| \leq c/(1+|\lambda|)$, $\lambda \in (-\infty, 0]$. By $\mathcal{C}^k$, $k \in \mathbb{Z}_+$, we denote the domain of A^k with norm $\|x\|_{\mathcal{C}^k} = \|A^k x\|_X$, $x \in \mathcal{C}^k$.

Let $0 < \theta < 1$ and $1 \leq q \leq \infty$. For pare complex Banach spaces $\{X, Y\}$ we define the interpolation space $(X,Y)_{\theta,q} = \big\{v \in X + Y : \|v\|_{(X,Y)_{\theta,q}} < \infty\big\}$ with norm

$$\|v\|_{(X,Y)_{\theta,q}} = \begin{cases} \Big(\displaystyle\int_0^\infty \big[t^{-\theta}\mathcal{K}(t,v;X,Y)\big]^q \frac{dt}{t}\Big)^{1/q} & : \quad q < \infty, \\ \displaystyle\sup_{0<t<\infty} t^{-\theta}\mathcal{K}(t,v;X,Y) & : \quad q = \infty, \end{cases}$$

where $\mathcal{K}(t,v;X,Y) = \inf\limits_{v=x+y} \big(\|x\|_X + t\,\|y\|_Y\big)$.

Let $m, k \in \mathbb{Z}$, $m \geq 0$. For all $\alpha \in \mathbb{C}$, such that $-m < \operatorname{Re}\alpha \leq \sigma - m$, $0 < \sigma < k$, and all $x \in \big(X, \mathcal{C}^k\big)_{\sigma/k,1}$ is defined the operator

$$A^\alpha_\sigma x = \frac{\Gamma(k)}{\Gamma(\alpha+m)\Gamma(k-m-\alpha)} \int_0^\infty t^{\alpha+m-1} A^{k-m}(A+tI)^{-k} x\, dt,$$

that assumes a closure A^α in X. The domain $\mathcal{C}^\alpha$ of A^α we consider as Banach space with norm $\|x\|_{\mathcal{C}^\alpha} = \|A^\alpha x\|_X$, $x \in \mathcal{C}^\alpha$. Thus, A^α and $\mathcal{C}^\alpha$ are defined for all numbers $\alpha \in \mathbb{C}$. The operator A^α is continuous for $\operatorname{Re}\alpha < 0$ and $A^\alpha : \mathcal{C}^\alpha \to X$ is isomorphic mapping for $\operatorname{Re}\alpha > 0$. The subspace $\mathcal{C}^k$ is dense in X and since $\mathcal{C}^k \subset \big(X, \mathcal{C}^k\big)_{\sigma/k,1} \subset \mathcal{C}^\alpha \subset X$, the domain $\mathcal{C}^\alpha$ is dense in X. For any $\alpha, \beta \in \mathbb{C}$ the next equalities $A^\alpha A^\beta = A^\beta A^\alpha = A^{\alpha+\beta}$ are held and $A^\beta : \mathcal{C}^{\alpha+\beta} \to \mathcal{C}^\alpha$ is isomorphic mapping. In particular, $\mathcal{C}^{\alpha+\beta} \subset \mathcal{C}^\alpha$ and $\mathcal{C}^\infty \subset \bigcap_\beta \mathcal{C}^{\alpha+\beta} \subset \mathcal{C}^\infty$, where we put $\mathcal{C}^\infty := \bigcap_{k\in\mathbb{Z}_+} \mathcal{C}^k$.

For any $x \in \mathcal{C}^\infty = \bigcap_{k\in\mathbb{Z}_+} \mathcal{C}^{\alpha+k}$ we put $x_k := A^k x \in \mathcal{C}^\alpha$. For pare numbers $1 \leq q \leq \infty$, $\nu > 0$ we define the spaces

$$\mathcal{E}^\nu_q(\mathcal{C}^\alpha) = \Big\{x \in \mathcal{C}^\infty : \|x\|_{\mathcal{E}^\nu_q(\mathcal{C}^\alpha)} = \Big(\sum\nolimits_{k\in\mathbb{Z}_+} \nu^{-kq} \|x_k\|^q_{\mathcal{C}^\alpha}\Big)^{1/q} < \infty\Big\},$$

$$\mathcal{E}^\nu_\infty(\mathcal{C}^\alpha) = \Big\{x \in \mathcal{C}^\infty : \|x\|_{\mathcal{E}^\nu_\infty(\mathcal{C}^\alpha)} = \sup_{k\in\mathbb{Z}_+} \nu^{-k} \|x_k\|_{\mathcal{C}^\alpha} < \infty\Big\}.$$

We investigate the interpolation properties of such spaces and approximation spaces generated by them. We consider the applications to the regular elliptic boundary problems where exponential type vectors coincide with root vectors.

Remark. *If* $\operatorname{Re}\alpha \leq 0$, *then* $\mathcal{C}^\alpha = X$ *and* $\mathcal{E}^\nu_\infty(X)$ *consists of exponential type* ν *vectors of* A.

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