THE CORRESPONDENCE BETWEEN THE FORMAL DOUBLE POWER SERIES AND TWO-DIMENSIONAL g-FRACTION WITH NONEQUIVALENT VARIABLES

R.I. DMYTRYSHYN AND O.S. MANZIY

One of the methods of expanding the functions of two variables, given by the formal double power series, into the two-dimensional continued fractions is the construction of corresponding two-dimensional continued fractions [1].

We consider the two-dimensional g-fraction with nonequivalent variables

$$\frac{s_0}{\Phi_0(z_1) + \mathop{\mathrm{D}}\limits_{n=1}^{\infty} \frac{g_{0n}(1-g_{0,n-1})z_2}{\Phi_n(z_1)}}, \quad \Phi_l(z_1) = 1 + \mathop{\mathrm{D}}\limits_{k=1}^{\infty} \frac{g_{kl}(1-g_{k-1,l})z_1}{1}, \ l \geq 0, \tag{1}$$

where $s_0 > 0,\ g_{00} = 0,\ 0 < g_{kl} < 1,\ k \geq 0,\ l \geq 0,\ k+l > 0,\ \mathbf{z} = (z_1, z_2) \in \mathbb{C}^2$, which is generalization of the continued g-fraction [2].

The correspondence between the two-dimensional g-fraction with nonequivalent variables (1) and the formal double power series

$$\sum_{k,l=0}^{\infty} (-1)^{k+l} s_{kl} z_1^k z_2^l, \tag{2}$$

where $s_{kl} \in \mathbb{R},\ k \geq 0,\ l \geq 0,\ \mathbf{z} \in \mathbb{C}^2$, means that the expansion of each nth approximant, $n \geq 1$, into the formal double power series coincides with the given series for all homogeneous polynomials to the degree $\nu_n - 1$ inclusively. The ν_n is called the order of correspondence.

We prove the following theorem:

Theorem 1. *For the two-dimensional g-fraction with nonequivalent variables* (1) *there exists the unique formal double power series of form* (2) *to which this fraction will correspond. The order of correspondence is $\nu_n = n$.*

The following theorem deals with the convergence of corresponding two-dimensional g-fraction with nonequivalent variables to formal double power series.

Theorem 2. *The two-dimensional g-fraction with nonequivalent variables* (1) *converges in the domain $\mathcal{Q} = \{\mathbf{z} \in \mathbb{C}^2 : \ |z_1| < 1/2,\ |z_2| < 1/2\}$ to function $g(\mathbf{z})$ which is holomorphic in this domain. The sum of the formal double power series* (2)*, which corresponds to the two-dimensional g-fraction with nonequivalent variables* (1)*, has the same value as this fraction in the domain $\mathcal{Q}$.*

References

[1] D. I. Bodnar, Branched continued fractions, Naukova Dumka, Kiev, 1986 (in Russian)
[2] H. S. Wall, Analytic theory of continued fractions, Van Nostrand, New York, 1948

Department of Economical Cybernetics, Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Shevchenko Str., 57, Ukraine
E-mail address: dmytryshynr@hotmail.com

Institute of Applied Mathematics and Fundamental Sciences, Lviv Polytechnic National University, Lviv, S. Bandera Str., 12, Ukraine
E-mail address: lesly@ukr.net