

**CENTRAL STATES ON THE INFINITE GENERAL LINEAR  
GROUP OVER A FINITE FIELD**

A.V. DUDKO

Let  $\mathbb{F}_q$  be a finite field, where  $q = p^n$ ,  $p$  is prime and  $n \in \mathbb{N}$ . By definition, infinite general linear group over  $\mathbb{F}_q$  denoted by  $G(\infty) = GL(\infty, \mathbb{F}_q)$  is the group of all invertible infinite matrices of the form  $I + A$ , where  $A$  has only finitely many nonzero entries from  $\mathbb{F}_q$ . The description of indecomposable characters on  $GL(\infty, \mathbb{F}_q)$  is given by Scudlarek [1].

**Theorem 1.** *Let  $\mathbb{F}_q^*$  be the multiplicative group of the field  $\mathbb{F}_q$ . A function  $\chi$  on  $GL(\infty, \mathbb{F}_q)$  is an indecomposable character iff either  $\chi(g) = \delta_{I, g}$  or there exists  $k \in \mathbb{N} \cup \{0\}$  and a homomorphism  $\nu : \mathbb{F}_q^* \rightarrow \mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$ , such that  $\chi(g) = q^{-k \cdot \text{rank}(I-g)} \nu(\det(g))$  for any  $g \in GL(\infty, \mathbb{F}_q)$ .*

The notion of a central state with respect to a subgroup is a natural generalization of a character. Namely, let  $\pi$  be a unitary representation of a group  $G$  and  $H \subset G$  be a subgroup. Denote  $\mathcal{M}_\pi$  the  $W^*$ -algebra, generated by the operators of representation  $\pi$ . A state  $\varphi$  on  $\mathcal{M}_\pi$  is called  $H$ -central, if  $\varphi(\pi(gh)) = \varphi(\pi(hg))$  for all  $g \in G, h \in H$ . Introduce some subgroups of  $G(\infty)$ , which we will use to state the main result. Let  $r, s, t \in \mathbb{Z}_+$ . Denote

$$G_r(\infty) = \left\{ g \in G(\infty) : g = \begin{bmatrix} I_r & 0 \\ 0 & *_\infty \end{bmatrix} \right\},$$

$$\tilde{G}(r, s, t) = \left\{ g \in G(\infty) : g = \begin{bmatrix} *_{r_1} & * & * & * \\ 0 & *_{s_1} & * & 0 \\ 0 & 0 & *_{t_1} & 0 \\ 0 & 0 & * & *_\infty \end{bmatrix} \right\},$$

$$\tilde{H}_0(r, s, t) = \left\{ g \in \tilde{G}(r, s, t) : g = \begin{bmatrix} I_r & 0 & * & * \\ 0 & I_s & 0 & 0 \\ 0 & 0 & I_t & 0 \\ 0 & 0 & * & I_\infty \end{bmatrix} \right\},$$

where  $*$  stands for arbitrary matrix of corresponding size and  $*_k$  stands for invertible  $k \times k$  matrix over  $\mathbb{F}_q$ . We prove the following.

**Theorem 2.** *Let  $\pi$  be a factor representation of  $G(\infty)$ . Assume, there exists a  $G_l(\infty)$ -central state on  $\mathcal{M}_\pi$  for some  $l$ . Then there exist  $r, s, t \in \mathbb{Z}_+$  and a factor representation  $\tilde{\pi}$  of the group  $\tilde{G}(r, s, t)$ , trivial on the subgroup  $\tilde{H}_0(r, s, t)$ , such that  $\pi$  is quasiequivalent to the representation  $\text{Ind}_{\tilde{G}(r, s, t)}^{G(\infty)} \tilde{\pi}$ .*

REFERENCES

- [1] Skudlarek H. L., *Die unzerlegbaren Charaktere einiger diskreter Gruppen*, Math. Annal., **233** (1976), pp. 213-231.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TORONTO, CANADA  
E-mail address: artem.dudko@utoronto.ca