THE STABILITY OF SOLUTIONS OF THE CAUCHY PROBLEM FOR A DIFFERENTIAL-OPERATOR EQUATION IN A HILBERT SPACE

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We consider the equation

$$\varepsilon y^{(4)}(t) + (A+E)y''(t) + Ay(t) = 0, \quad t \in [0,\infty), \tag{1}$$

where A is a selfadjoint lover semibounded operator in a Hilbert space H and $\varepsilon > 0$. In the case where $H = L_2[a,b]$ and A is some selfadjoint extension of the minimal operator generated by the expression $-d^2/dx^2$, this equation is the equation of the dynamics of a stratified fluid.

A vector-valued function $y(t)$ is called a solution of equation (1) if it is four times strongly differentiable in H, $y(t), y''(t) \in D(A)$ (the domain of A) and $y(t)$ satisfies (1). A solution is called stable if

$$\sup_{\substack{t\in[0,\infty)\\ k=0,1,2,3}} \|y^{(k)}(t)\| < \infty.$$

We investigate the stability of solutions in terms of the distribution of the spectrum of the operator A. We have received necessary and sufficient conditions of the stability of solutions of the Cauchy problem for this equation. If each solution is stable, then we say that the equation is stable.

Let $\sigma(A)$ be the spectrum of the operator A, and let

$$\gamma = \min\bigl\{0, \inf\nolimits_{\|f\|=1}(Af,f)\bigr\}.$$

Theorem. *For arbitrary $f_0, f_1, f_2 \in D(A)$, $f_3 \in D(A-\gamma E)^{1/2}$ there exists a unique solution of equation* (1) *which satisfies the initial conditions $y_k(0) = f_k$, $k = 0,1,2,3$. Equation* (1) *is stable if and only if*

$$(-\infty,0]\cap\sigma(A) = \varnothing \quad \textit{for} \quad 0<\varepsilon<1,$$

$$\Bigl((-\infty,0]\cup\bigl[2\varepsilon-1-2\sqrt{\varepsilon^2-\varepsilon}, 2\varepsilon-1+2\sqrt{\varepsilon^2-\varepsilon}\bigr]\Bigr)\cap\sigma(A) = \varnothing \quad \textit{for} \quad \varepsilon\ge 1.$$

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