ON THE LAST PROGRESS OF COHOMOLOGICAL INDUCTION IN THE PROBLEM OF CLASSIFICATION OF THE LIE GROUPS REPRESENTATIONS

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The extensions and globalizations of modules of Harish-Chandra are exposed to obtain a theory of representations that includes the cases of non compact type of irreducible representations studied in the program of Vogan. For this expands it an infinitesimal character of weight λ_L-$\rho(\mathfrak{u})$ anti-dominat of a representation $H_c^{p,q}(X, \nu)$ with ν defined by X = G/L.

Lemma 1. *Considering the continuous* $\mathfrak{u}$*-cohomology and the generalization of the topology on holomorphic complex vector bundles of Frechet* $E_\eta \longrightarrow G/L$ *one has that* $H_{ct}^{\bullet}(\mathfrak{u}, I^{\infty}(\eta)) = H_{ct}^{\bullet}\left(G/L, O_{\mathfrak{q}}(E_\eta)\right)$.

Corollary. *Be* $Z = K/L \cap K$ *a s-dimensional compact complex submanifold of the n-dimensional complex manifold* $X = G/L$. *Be r = n - 1 the codimension of Z to X. Assume you that V is a* ($\mathfrak{q}$, L)*- module of infinitesimal character* $\lambda_L \in \mathfrak{h}*$ *and that V is the minimal globalization of the* $(\mathfrak{q}, L \cap K) -$ *module. Be* $\lambda_L - \rho(\mathfrak{u})$ *weakly anti-dominant to* $\mathfrak{u}$, *this is that -* $\lambda_L + \rho(\mathfrak{u})$ *is weakly anti-dominant. Then (i)* $H_c^{0,q}(X, \nu) = 0$, *under q = r, (ii) If* $L = L_{MAX}$ *and V is inductive representation of L then* $H_c^{0,r}(X, \nu)$ *is irreducible or zero, (iii) If the module of Harish-Chandra of V it admits a invariant Hermitian form, then the module of Harish-Chandra of* $H_c^{0,r}(X, \nu)$, *it admits a Hermitian form, (iv) If the module of Harish-Chandra of V is unitary then the module of Harish-Chandra of* $H_c^{0,r}(X, \nu)$,*is unitary .*

Theorem 1. *Be* η *denoted for* $H_{ct}^{t}\left(L(w), O_{\mathfrak{q}}(E_{\gamma\nu})\right)$, *a representation of infinite dimension of* $(\mathfrak{q}, L)$ *and be* $E_\eta \longrightarrow G(w)$ *the associated homogeneous vector bundle. Then the operator* $\overline{\partial}$ *to the Dolbeault complex* $A\left(G(w), E_\eta \otimes \left(\wedge^{q+1}\mathfrak{u}\right)^*\right)^L$ *is of closed range. Then the cohomologies* $H^q\left(G(w), O_{\mathfrak{q}}(E_\eta)\right) = 0\ \forall\ q \neq s$ *are admisible G-modules of Frechet. Their underlying modules of Harish-Chandra are functors of Zuckerman* $A^{s+t}(G, X, \mathfrak{k}, \gamma_\nu) = A(G, L, \mathfrak{q}, \eta)$. E_η *have infinitesimal character* $\eta_{L,\lambda}$ *and trivial action* $\mathfrak{u}$.

Question. *Is of close range* $\overline{\partial}$ *on the images of the Dolbeault complexes under the Penrose transform to* λ *appropriate?*

References

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