

REGULARITY FOR SET MULTIFUNCTIONS IN HAUSDORFF TOPOLOGY

ALINA CRISTIANA GAVRILUT

As it is well-known, regularity is an important property of continuity, which connects measure theory and topology, approximating general Borel sets by more tractable sets, such as compact and/or open sets.

In this paper we study different types of regularity for monotone uniformly autocontinuous set multifunctions μ defined on a ring of subsets of a locally compact, Hausdorff space T and taking values in $\mathcal{P}_f(X)$, the family of closed, nonvoid sets of a real normed space X , family which is endowed with the Hausdorff pseudometric. Particularly, if the monotone uniformly autocontinuous set multifunction is a multisubmeasure, we establish relationships with other continuity properties such as exhaustivity and order continuity. Extensions of multisubmeasures by preserving the regularity are obtained. Also, if f is a $\tilde{\mu}$ -totally measurable real valued set function, where μ is a monotone exhaustive uniformly autocontinuous set multifunction, a Lusin type theorem is discussed.

Theorem 1. *i) μ is R -regular if and only if it is R_l -regular and R_r -regular.*

ii) μ is R' -regular if and only if it is R'_l -regular and R'_r -regular.

iii) If μ is R'_l -regular, then μ is R_l -regular.

iv) If μ is R'_r -regular, then μ is R_r -regular.

v) If μ is R' -regular, then μ is R -regular.

vi) If \mathcal{C} is the ring (or the δ -ring) generated by the compact sets or by the compact, G_δ sets, then μ is R'_l -regular if and only if μ is R'_r -regular if and only if it is R' -regular.

Theorem 2. *If μ is exhaustive and R_l -regular, then it is R' -regular.*

Corollary 1. *Suppose $\mathcal{C} = \mathcal{B}_0$ or \mathcal{B} and μ is exhaustive. The following statements are equivalent:*

i) μ is R'_l -regular;

ii) μ is R'_r -regular;

iii) μ is R' -regular;

iv) μ is R_l -regular;

v) μ is R -regular.

Theorem 3. *Suppose $\mu : \mathcal{B}_0 \rightarrow \mathcal{P}_f(X)$.*

i) If μ is order-continuous, then μ is R' -regular.

ii) If μ is a multisubmeasure, then μ is order-continuous if and only if it is R' -regular.

Theorem 4. *If X is a Banach space and if $\nu : \mathcal{B}_0 \rightarrow \mathcal{P}_{bf}(X)$ is a R' -regular Baire multisubmeasure, then ν uniquely extends to a R' -regular Borel multisubmeasure $\mu : \mathcal{B} \rightarrow \mathcal{P}_{bf}(X)$.*

Question. *Is Lusin's theorem valid for exhaustive monotone uniformly autocontinuous set multifunctions?*

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"AL.I. CUZA" UNIVERSITY,, FACULTY OF MATHEMATICS,, BD. CAROL I, NO 11,, IAȘI-700506
E-mail address: gavrilut@uaic.ro